M2.1 Quadratic Sequences

Before you start

You should be able to:

- derive and use an expression for the nth term of an arithmetic sequence
- evaluate a quadratic expression given a positive value of the variable.

Why do this?

Quadratic sequences are used when solving the complex equations which describe how weather systems evolve.

Objective

- Derive and use an expression for the nth term of a quadratic sequence.

Get Ready

1 Find an expression in terms of n for the nth term of this arithmetic sequence:
3 5 7 9

2 Work out the value of $5n^2$ when $n = 2$.

3 Work out the value of $n^2 - 3n - 5$ when $n = 3$.

Key Points

- The nth term of a quadratic sequence has the form $an^2 + bn + c$ where a, b and c are numbers.
- The second differences of the terms in a quadratic sequence are constant and equal to $2a$.
- The sequence of square numbers starts 1, 4, 9, 16…
- The nth term of this sequence is n^2. This is the simplest quadratic sequence.

Example 1

The first 4 terms of a quadratic sequence are: 2 5 10 17

Find an expression in terms of n for the nth term.

As a first step, compare this sequence to 1 4 9 16

Comparing to 1 4 9 16 the terms of the quadratic sequence are one more.
The required expression for the nth term is $n^2 + 1$.

Exercise 2A

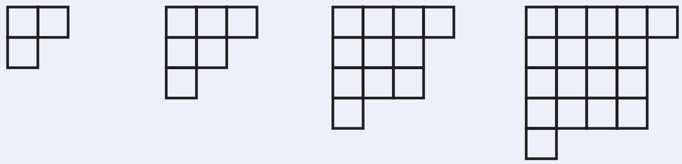
C

1 Find an expression in terms of n for the nth term of the quadratic sequences which start:

a	2	8	18	32
b	0	3	8	15
c	4	7	12	19
d	0	1	4	9
e	1	$1 + 3$	$1 + 3 + 5$	$1 + 3 + 5 + 7$
f	1×2	2×3	3×4	4×5

C

2 Here is a pattern made from centimetre squares.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

a Write down an expression in terms of n for the number of centimetre squares in pattern n.

b Is there a pattern in the sequence which has 170 centimetre squares?
Give a reason for your answer.

3 **a** Find an expression in terms of n for the nth term of the arithmetic sequence:

3 5 7 9

The sequence:

9 25 49 81

is obtained from squaring each term of the arithmetic sequence.

b Find an expression in terms of n for the nth term of this sequence.

B

4 **a** Write down an expression for the nth term of the sequence:

1 4 7 10

b Write down an expression for the nth term of the sequence.

1 16 49 100

c Write down an expression for the nth term of the sequence:

2 18 52 104

Example 2 Find an expression in terms of n for the nth term of the quadratic sequence:

-1 1 7 17 31

first differences: 2 6 10 14

second differences: 4 4 4

Work out the first differences $1 - -1 = 2$, $7 - 1 = 6$, $17 - 7 = 10$ and so on.

Then work out the second differences (the differences of the first differences): $6 - 2 = 4$, $10 - 6 = 4$, $14 - 10 = 4$

If the second differences are constant then the sequence is quadratic.

The expression for the nth term will be of the form $an^2 + bn + c$ where a, b and c are numbers.

a = Second difference $\div 2 = 2$, so the expression for the nth term is $2n^2 + bn + c$

To find the values of b and c, work out the difference between the terms of the sequence and $2n^2$ as shown in the table.

n	1	2	3	4	5
Term	−1	1	7	17	31
$\mathbf{2n^2}$	2	8	18	32	50
Term $-$ $\mathbf{2n^2}$	−3	−7	−11	−15	−19

← These are the terms of the quadratic sequence

← $31 - 2 \times 5^2 = -19$

The sequence −3 −7 −11 −15 −19 has nth term $-4n + 1$.
So the required expression for the nth term of the sequence −1 1 7 17 31 is $2n^2 - 4n + 1$.

Exercise 2B

1 Find the next term and an expression for the nth term of these quadratic sequences:

a	2	6	12	20	30
b	0	2	6	12	20
c	3	7	13	21	31
d	13	17	23	31	41
e	−4	0	10	26	48
f	3	5	8	12	17

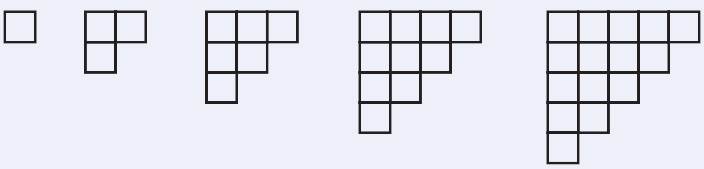
2 Show that 862 is the 20th term of the quadratic sequence:
7 16 29 46 67

3 Show that 5005 is the 50th term of the quadratic sequence:
7 13 23 37 55

4 Here are the first 5 terms of a quadratic sequence:
4 15 30 49 72
Show that there are no prime numbers in the quadratic sequence.

5 Here are the first 4 terms of a quadratic sequence:
3 9 17 27 39
Jim says that 161 is a term of this sequence.
a Is Jim correct? Give a reason for your answer.
Lizzie says that all of the terms are odd numbers.
b Is Lizzie correct? Give a reason for your answer.

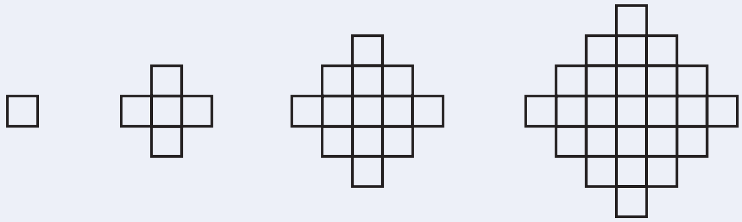
6 Here is a sequence of patterns made of centimetre squares.



Find the number of centimetre squares in the 100th pattern.

A

7 Here is a sequence of patterns made of centimetre squares.



Find an expression in terms of n for the number of centimetre squares in the n^{th} pattern.

8 The 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of a quadratic sequence are:

34 37 42 49

Find an expression, in terms of n, for the nth term of this sequence.

9 Here are the first four terms of a quadratic sequence:

4 9 18 x

a Find the value of x.

b Find an expression for the nth term of the sequence.

10 At a party, everyone shakes hands with everyone else. So when there are 4 people at a party there are 6 handshakes. Find an expression in terms of n for the number of handshakes when there are n people at the party.

Review

- The sequence of square numbers begins 1, 4, 9, 16, 25 and the nth term is n^2.
- The nth term of a quadratic sequence can be written as $an^2 + bn + c$.
- The second differences of a quadratic sequence are constant and equal to $2a$.

Answers

Chapter 2

M2.1 Get Ready answers

1 $2n + 1$
2 20
3 -5

Exercise 2A answers

1 a $2n^2$ b $n^2 - 1$ c $n^2 + 3$
d $(n - 1)^2$ e n^2 f $n(n + 1)$
2 a $n^2 + 2$
b No, because $12^2 + 2 = 146$ and $13^2 + 2 = 171$
3 a $2n + 1$ b $(2n + 1)^2$
4 a $3n - 2$ b $(3n - 2)^2$ c $(3n - 2)^2 + n$

Exercise 2B answers

1 a 42, $n^2 + n$ b 30, $n^2 - n$
c 43, $n^2 + n + 1$ d 53, $n^2 + n + 11$
e 76, $3n^2 - 5n - 2$ f 23, $\frac{1}{2}(n^2 + n + 4)$
2 nth term is $2n^2 + 3n + 2$.
When $n = 20$, $2n^2 + 3n + 2 = 800 + 60 + 2 = 862$
3 nth term is $2n^2 + 5$.
When $n = 50$, $2n^2 + 5 = 2 \times 2500 + 5 = 5005$
4 nth term is $2n^2 + 5n - 3$ which factorises to $(2n - 1)(n + 3)$ so cannot be prime
5 a nth term is $n^2 + 3n - 1$, 11^{th} term is 153, 12^{th} term is 179, so no.
b Yes, because the first differences are even numbers and the first term is an odd number.
6 nth term $= \frac{1}{2}n^2 + \frac{1}{2}n$, 100^{th} term $= 5050$
7 $2n^2 - 2n + 1$
8 $n^2 - 6n + 42$
9 a 31 b $2n^2 - n + 3$
10 $\frac{n^2}{2} - \frac{n}{2}$